

PH 222-3A Spring 2010

ELECTRIC CHARGE

Lecture 1

Chapter 21

(Halliday/Resnick/Walker, Fundamentals of Physics 8th edition)

Chapter 21

Electric Charge

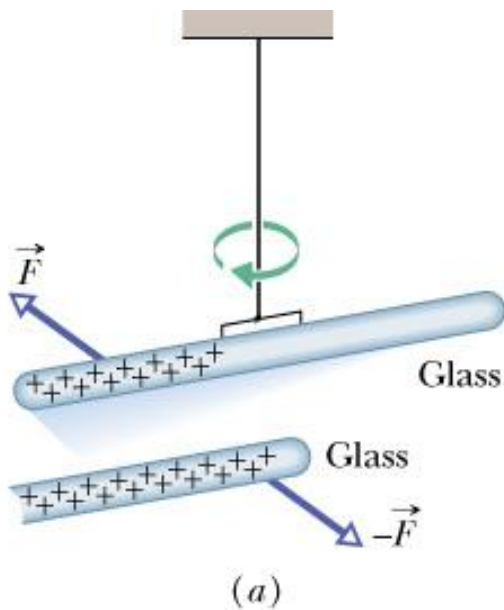
In this chapter we will introduce a new property of matter known as “electric charge” (symbol q).

We will explore the charge of atomic constituents.

Moreover, we will describe the following properties of charge:

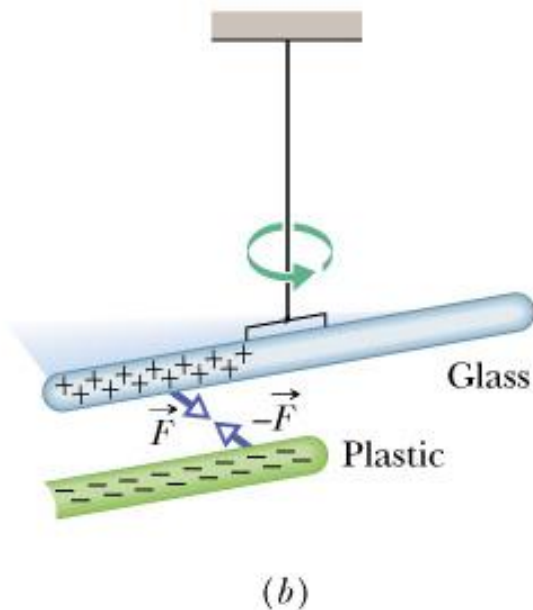
- Types of electric charge
- Forces among two charges (Coulomb's law)
- Charge quantization
- Charge conservation

Empirically it was known since ancient times that if amber is rubbed on cloth, it acquires the property of attracting light objects such as feathers. This phenomenon was attributed to a new property of matter called “electric charge.” (Electron is the Greek name for amber.) More experiments show that there are two distinct types of electric charge: **positive** (color code: red) and **negative** (color code: black). The names “positive” and “negative” were given by Benjamin Franklin.



When we rub a glass rod with silk cloth, both objects acquire electric charge. The sign on the charge on the glass rod is defined as **positive**.

In a similar fashion, when we rub a plastic rod with fur both objects acquire electric charge. The sign on the charge on the plastic rod is defined as **negative**.



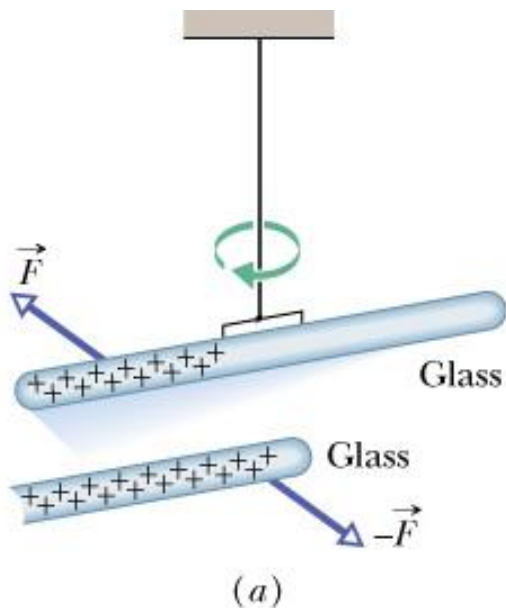
Q: Do we have enough information so as to be able to determine the sign of all other charges in nature? To answer this question we need one more piece of information.

Further experiments on charged objects showed that:

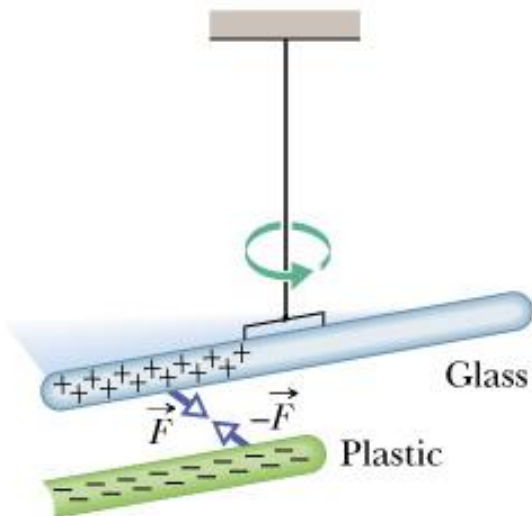
1. Charges of the same type (either both positive or both negative) repel each other (fig. a).
2. Charges of opposite type on the other hand attract each other (fig. b).

The force direction allows us to determine the sign of an unknown electric charge.

Charges of the same sign repel each other. Charges of opposite sign attract each other.



(a)



(b)

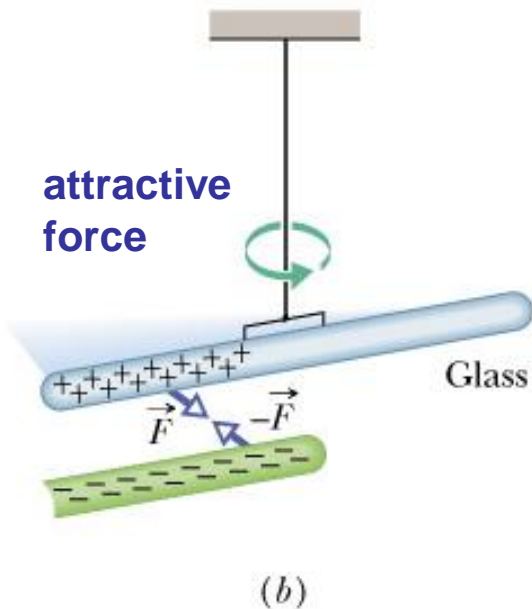
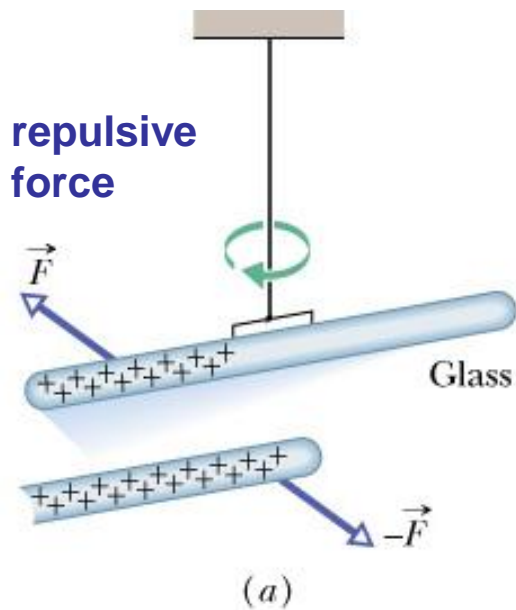
The recipe is as follows:

We charge a glass rod by rubbing it with silk cloth. Thus we know that the charge on the glass rod is positive. The rod is suspended in such a way so that it can keep its charge and also rotate freely under the influence of a force applied by charge with the unknown sign. We approach the suspended glass rod with the new charge whose sign we wish to determine.

Two outcomes are possible. These are shown in the figure to the left:

Fig. a: The two objects repel each other. We then conclude that the unknown charge has a **positive** sign.

Fig. b: The two objects attract each other. We then conclude that the unknown charge has a **negative** sign.



In Benjamin Franklin's day (18th century) it was assumed that electric charge is some type of weightless continuous fluid. Investigations of the structure of atoms by **Ernest Rutherford** at the beginning of the 20th century revealed how matter is organized and also identified that charge of its constituents.

Atoms consist of **electrons** and the **nucleus**.

Atoms have sizes $\sim 5 \times 10^{-10}$ m.

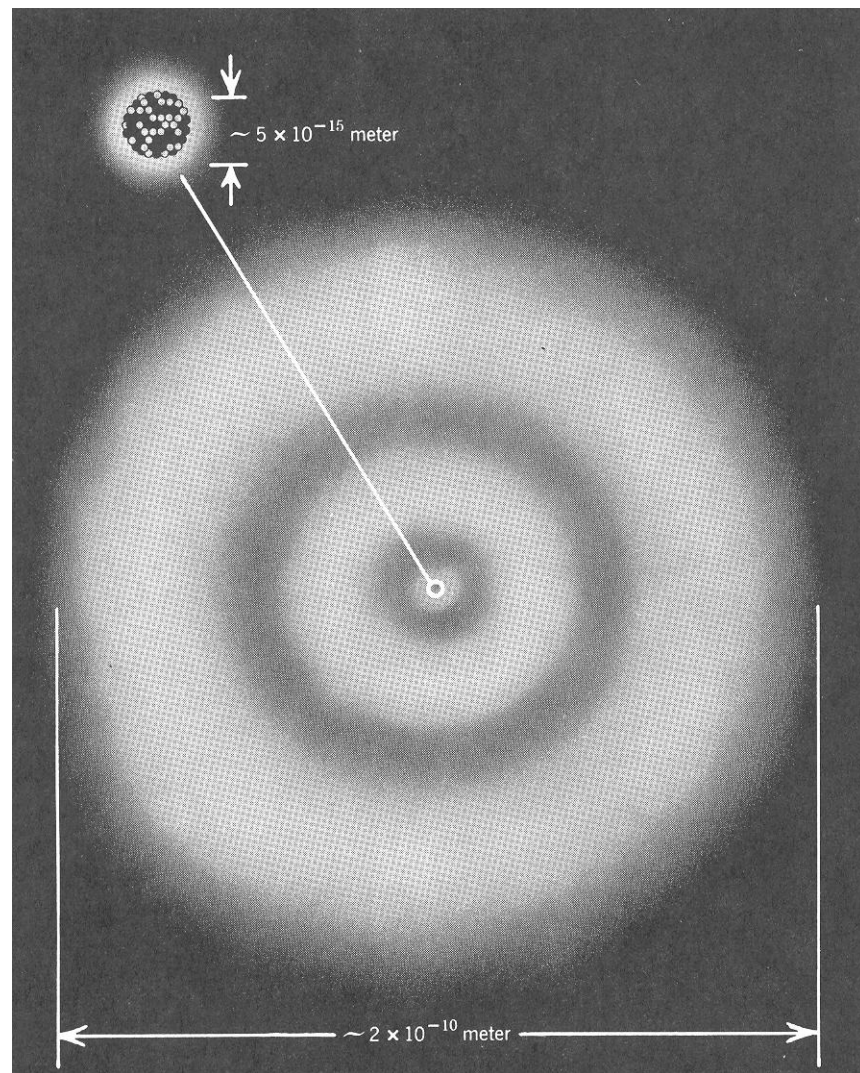
Nuclei have sizes $\sim 5 \times 10^{-15}$ m.

The nucleus itself consists of two types of particles: protons and neutrons.

The electrons are **negatively** charged.

The protons are **positively** charged.

The neutrons are **neutral** (zero charge).



Thus electric charge is a fundamental property of the elementary particles (electrons, protons, neutrons) out of which atoms are made.

Mass and Charge of Atomic Constituents

Neutron (n) : Mass $m = 1.675 \times 10^{-27}$ kg; Charge $q = 0$

Proton (p) : Mass $m = 1.673 \times 10^{-27}$ kg; Charge $q = +1.602 \times 10^{-19}$ C

Electron (e) : Mass $m = 9.11 \times 10^{-31}$ kg; Charge $q = -1.602 \times 10^{-19}$ C

Note 1: We use the symbols “-e” and “+e” for the electron and proton charge, respectively. This is known as the **elementary charge**.

Note 2: Atoms are electrically neutral. The number of electrons is equal to the number of protons. This number is known as the “**atomic number**” (symbol: Z). The chemical properties of atoms are determined **exclusively** by Z .

Note 3: The sum of the number of protons and the number of neutrons is known as the “**mass number**” (symbol: A).

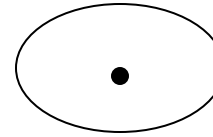
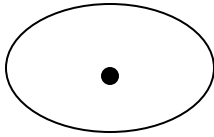
Notation: ${}_{92}^{235}\text{U}$

$Z = 92$ = number of protons/electrons

$A = 235$ = number of protons + neutrons

The atomic number $Z = 92$ defines the nucleus as that of a uranium atom.

The electrical forces between two neutral atoms tend to cancel



- Each electron in one atom is attracted by the protons in the nucleus of the other atom and simultaneously it is repelled by the equal number of electrons of that atom.
- Likewise, the electric forces between two neutral macroscopic bodies separated by some distance tend to cancel
- The cancellation of the electric forces between neutral macroscopic bodies explains why we do not see large electric attractions or repulsions between the macroscopic bodies, even though the electric forces between individual **e** and **p** are much stronger than the gravitational forces.

Problem.

What is the number of electrons and protons in a human body of mass 73 kg? The chemical composition of the body is roughly 70% oxygen, 20% carbon and 10% hydrogen (by mass).

Solution.

In the human body there are:

- (i) 51.1 kg O (73kg x 0.7 = 51.1kg)
- (ii) 14.6 kg C (73kg x 0.2 = 14.6kg)
- (iii) 7.3 kg H (73kg x 0.1 = 7.3kg)

We divide by atomic mass each element to find how many moles there are.

Number of moles are:

- (i) $51.1 / 0.016 = 3194 \text{ mol O}$
- (ii) $14.6 / 0.012 = 1217 \text{ mol C}$
- (iii) $7.3 / 0.001 = 7300 \text{ mol H}$

Each mol contains 6.02×10^{23} atoms. Each atom has a number of electrons(protons)=
=atomic number => total amount of electrons (protons) is:

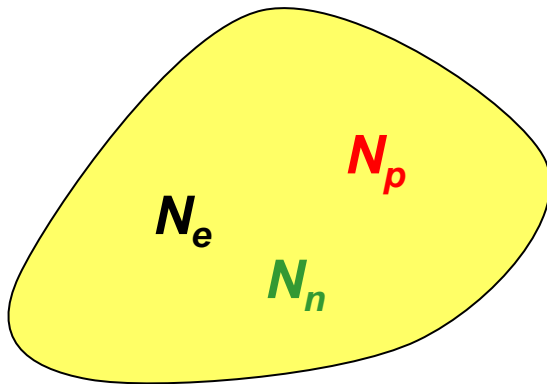
$$(6.02 \times 10^{23}) \times [(3194 \times 8) + (1217 \times 6) + (7300 \times 1)] = \boxed{2.4 \times 10^{28} \text{ electrons(protons)}} \quad 9$$

Charge Quantization

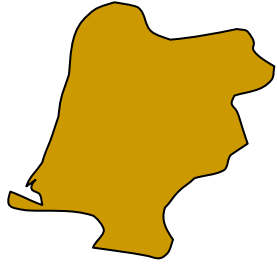
Now that we have identified the charge of the atomic constituents (electrons, protons, neutrons), it is clear that the net charge Q_{net} of an object that contains N_e electrons, N_p protons, and N_n neutrons is given by $Q_{\text{net}} = -eN_e + eN_p + 0N_n = e(N_p - N_e) = ne$.

Here $n = (N_p - N_e)$ and it is an integer. Thus the net charge is **quantized**.

This means that it cannot take any arbitrary value but only values that are multiples of the elementary charge e . The value of e is small and thus in many large-scale phenomena the "graininess" of electric charge is not apparent.

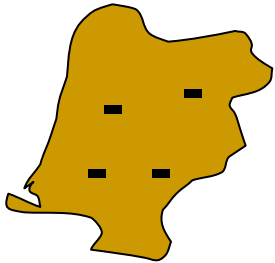


silk



glass rod

silk



glass rod

Conservation of Charge

Consider a glass rod and a piece of silk cloth (both uncharged) shown in the upper figure. If we rub the glass rod with the silk cloth we know that positive charge appears on the rod (see lower figure). At the same time an equal amount of negative charge appears on the silk cloth, so that the net rod-cloth charge is actually zero. This suggests that rubbing does not create charge but only transfers it from one body to the other, thus upsetting the electrical neutrality of each body. Charge conservation can be summarized as follows: In any process the charge at the beginning equals the charge at the end of the process.

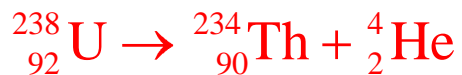
Net charge before = Net charge after

$$Q_i = Q_f$$

No exceptions of charge conservation have been found.

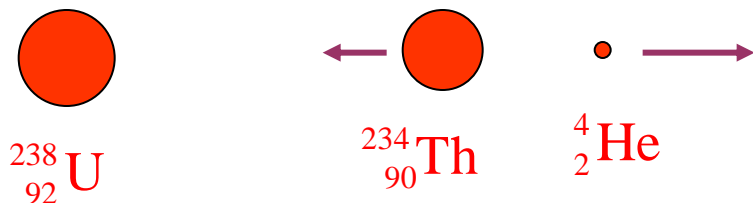
For example, charge is conserved in nuclear reactions.

An example is given below:



In this example, a parent nucleus of Uranium-238, which has 92 protons and $(238-92) = 146$ neutrons, decays into two products:

- i. A daughter Thorium-234 nucleus, which consists of 90 protons and $(234-90) = 144$ neutrons
- ii. A Helium-4 nucleus, which has 2 protons and 2 neutrons. The net charge before and after the decay remains the same, equal to $92e$.



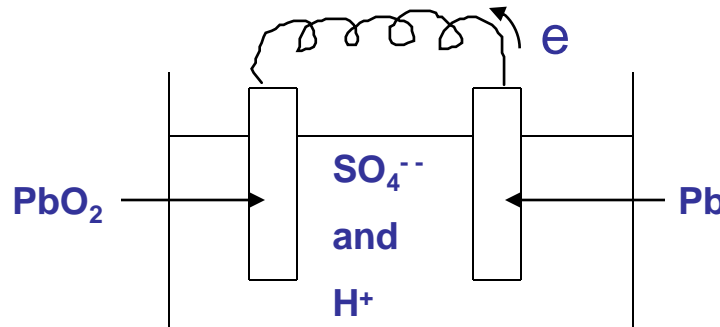
Charge Conservation

- The electric charge is a conserved quantity:

In any reaction involving charged particles, the total charges before and after the reaction are always the same. No reaction that creates or destroys net electric charge has ever been discovered.

Conservation of charge in chemical reactions in a lead–acid automobile battery.

The reaction releases electrons at the lead plate, electrons are absorbed at the lead dioxide plate.



Plates of lead and lead dioxide are immersed in an electrolytic solution of sulfuric acid.



Problem.

Consider the following hypothetical reactions involving the collision between a high energy proton (from an accelerator) and a stationary proton (in the nucleus of a hydrogen atom serving as a target).

- 1) $p + p \rightarrow n + n + \pi^+$
- 2) $p + p \rightarrow n + p + \pi^0$
- 3) $p + p \rightarrow n + p + \pi^+$
- 4) $p + p \rightarrow p + p + \pi^0 + \pi^0$
- 5) $p + p \rightarrow n + p + \pi^0 + \pi^-$

Where

p = proton

n = neutron

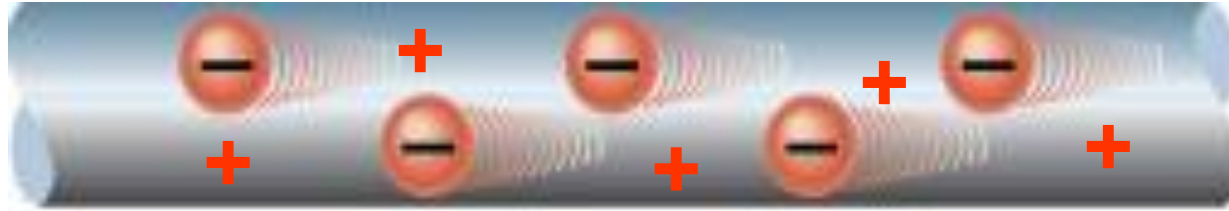
π^+ = positively charged pion (+e)

π^0 = neutral pion (e)

π^- = negatively charged pion (-e)

Which of these reactions are impossible, because they violate the conservation of charge?

- 1) $e + e \rightarrow 0 + 0 + e$ } Charge **is not** conserved, reaction is impossible
- 2) $e + e \rightarrow 0 + e + 0$ } Charge **is not** conserved, reaction is impossible
- 3) $e + e \rightarrow 0 + e + e$ } Charge **is** conserved
- 4) $e + e \rightarrow e + e + 0 + 0$ } Charge **is** conserved
- 5) $e + e \rightarrow 0 + e + 0 + (-e)$ } Charge **is not** conserved, reaction is impossible



Conductors and Insulators

Conductors are materials that allow charges to move freely through them.

Examples are: copper, aluminum, mercury.

Insulators are materials through which charges cannot move freely.

Examples are: plastic, rubber, glass, ceramics.

In conductors, one or more of the outermost electrons of the constituent atoms become free and move throughout the solid. These are known as conduction electrons. The conduction electrons leave behind positively charged atoms (known as ions). Only the negatively charged electrons are free to move inside a conductor. The positively charged ions are fixed in place.

Insulators do not have conduction electrons.

The electrons are held inside the metal in much the same way as particles of a gas are held inside a container.

=>

Electrons in metals form free electron gas

Conductors

Metals
gases
plasma

Electrolytes or liquid

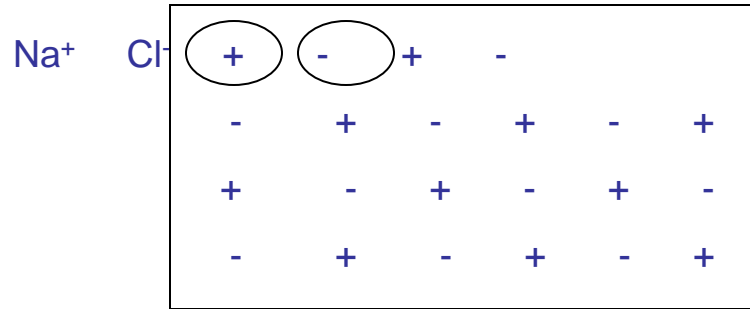
conductors containing
ions of impurity.
Solution of salt in water
 Na^+ , Cl^-

Ionized

Gases containing a
mixture of ions and
free electrons

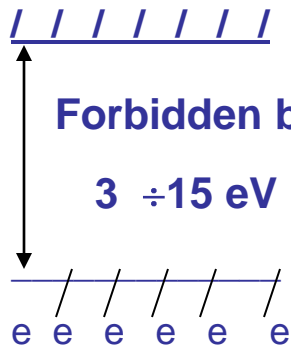
Ordinary gases are insulators. Ionization of a gas occurs whenever the gas molecules are subjected to large electric forces, that produce a sudden catastrophic ionization of the gas.

Insulator or Dielectric. Energy level diagram.



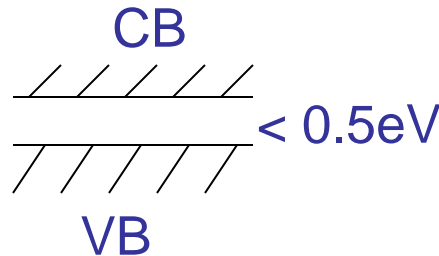
The ions of dielectric crystals hold their electrons strongly and so a sample doesn't contain free electrons. Free electrons can appear only when we ionize our ions and provide electrons with $KE > \text{Potential energy of attraction to the ions}$. This process can be described with the help of an energy level diagram.

Conduction band



Valence band

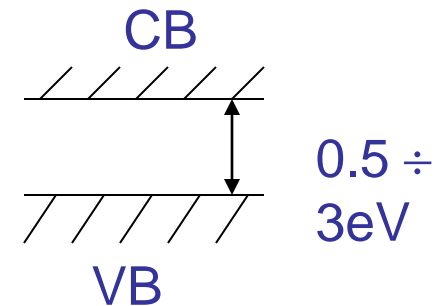
Insulator



Metal

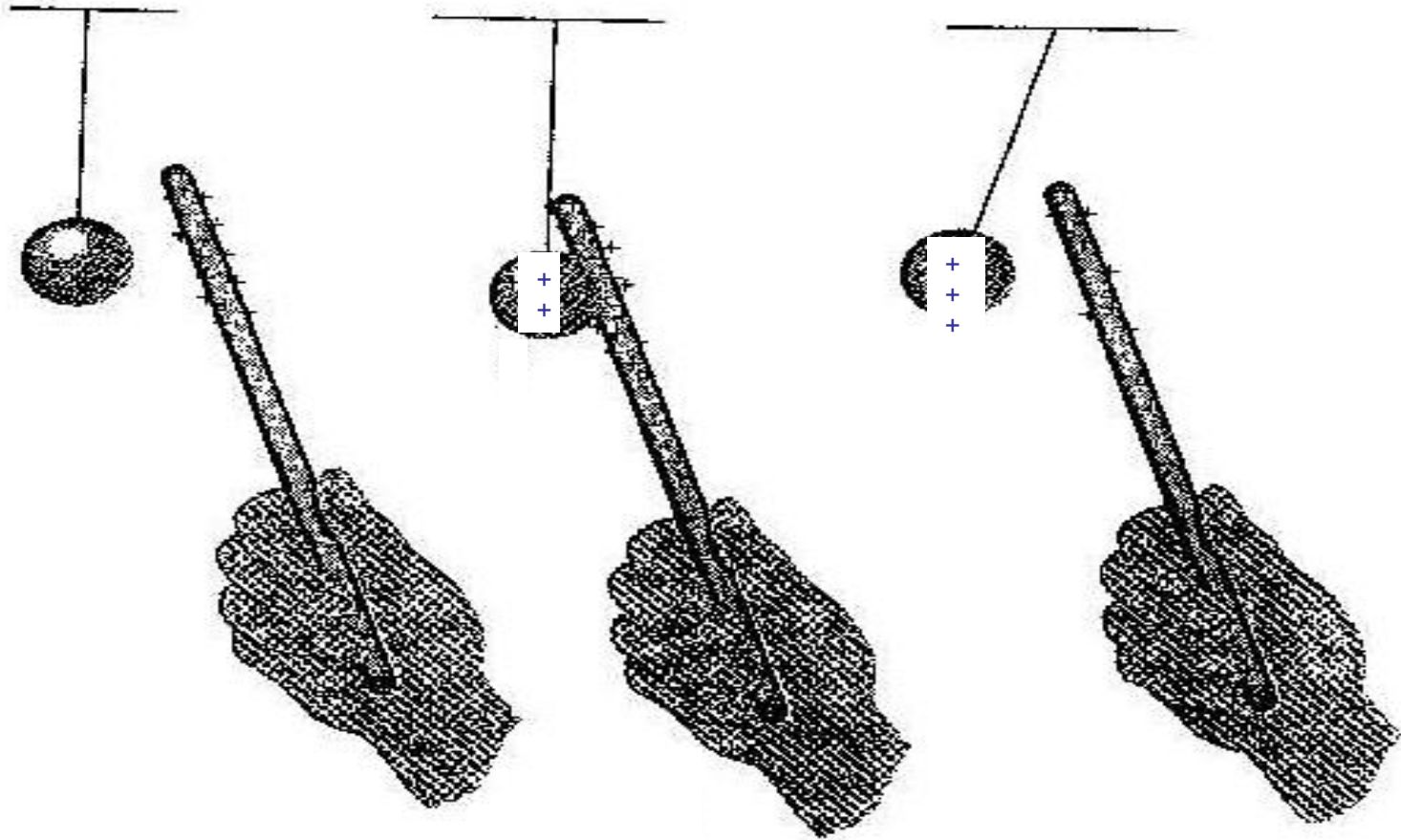
(conductor)

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$



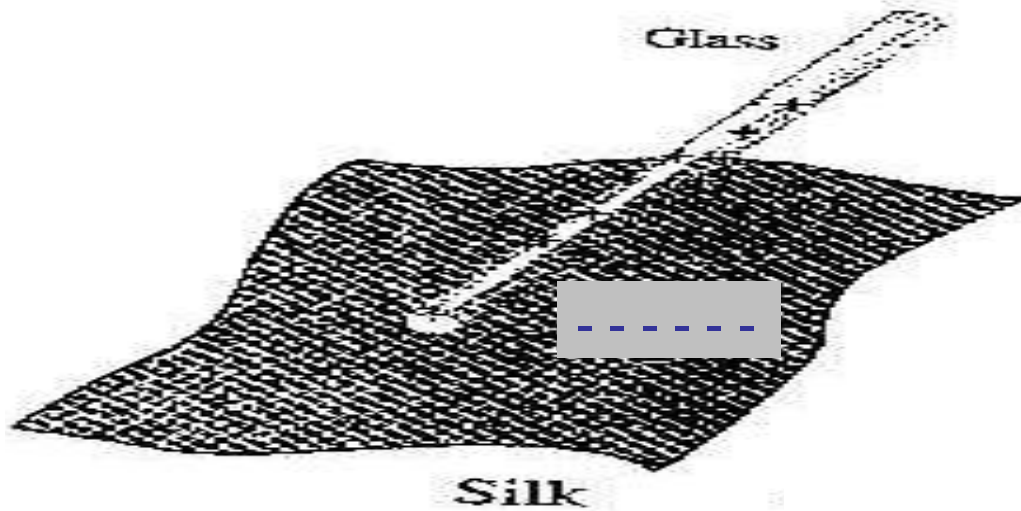
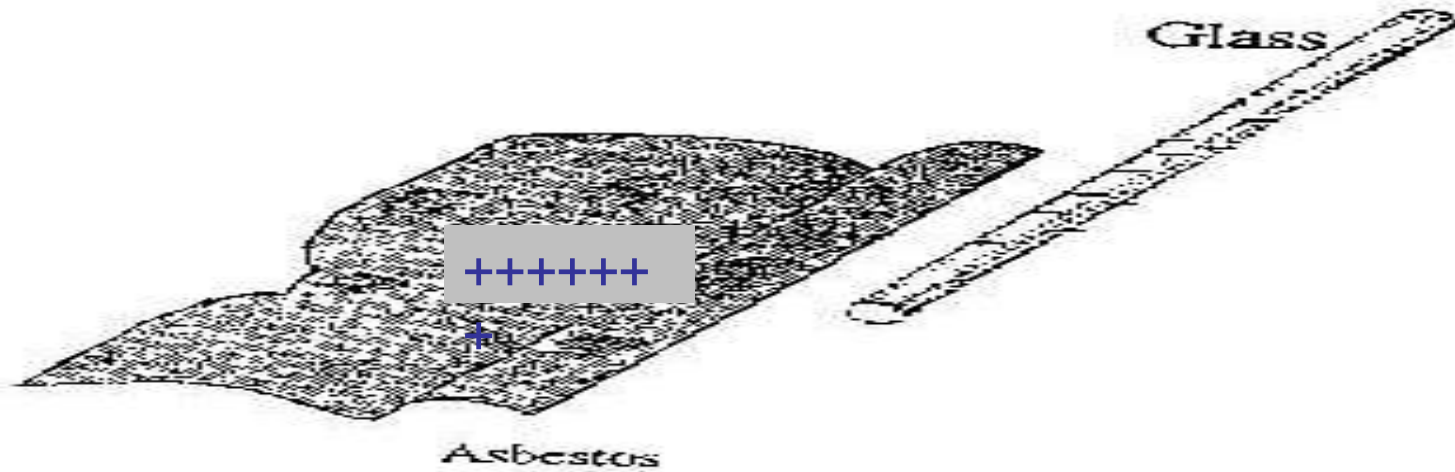
Semiconductor

Direct transfer of electrons



Charging a sphere of pith positively by direct transfer of electrons from ball to rod. This leaves the ball positively charged, and it is immediately repelled from the rod.

Frictional electricity. Charging by rubbing.



Electrons are transferred in contact from the asbestos to the glass, and from the glass to the silk.

Charging a Conductor by Induction

A conductor can be charged using the procedure shown in fig. *a* and fig. *b*. In fig. *a* a conductor is suspended using an insulating thread. The conductor is initially uncharged. We then approach the conductor with a negatively charged rod. The negative charges on the rod are fixed because plastic is an insulator.

These repel the conduction electrons of the conductor, which end up at the right end of the rod. The right end of the rod has an electron deficiency and thus becomes positively charged. In fig. *b* we provide a conducting path to ground (e.g., we can touch the conductor).

As a result, the electrons escape to the ground. If we remove the path to the ground and the plastic rod, the conductor remains positively charged.

Note 1: The induced charge on the conductor has the opposite sign of the charge on the rod.

Note 2: The plastic rod can be used repeatedly.

Fig. a

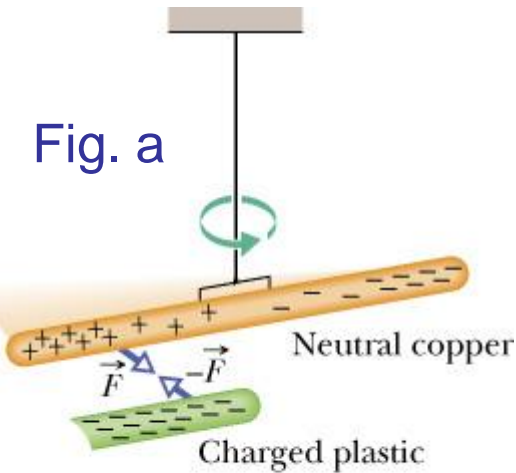
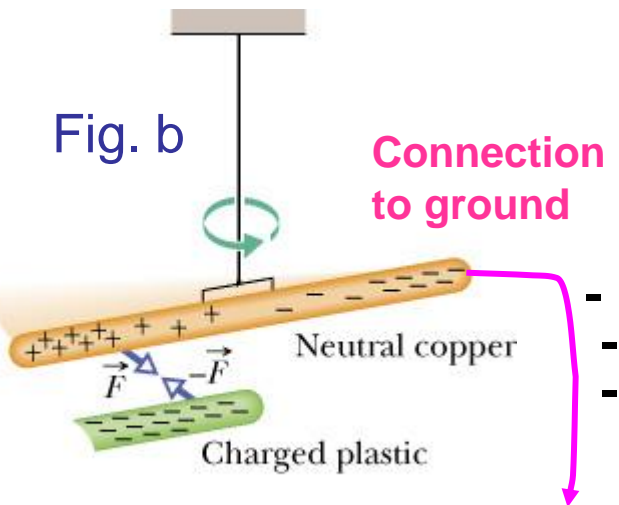
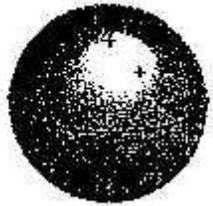


Fig. b

Connection
to ground



No matter what the shape of the conductor, excess charge always resides on its outer surface

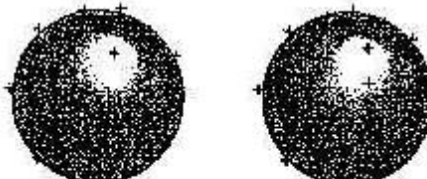
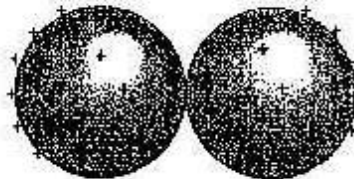
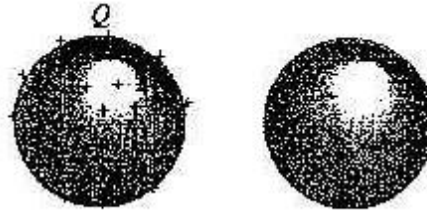


Conductor



Nonconductor

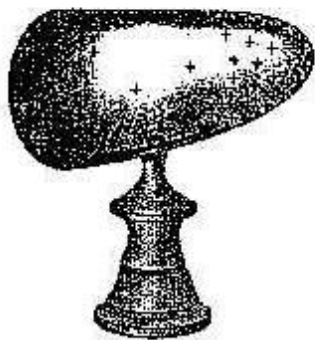
The distribution of charge placed on the surfaces of a conductor and a nonconductor.



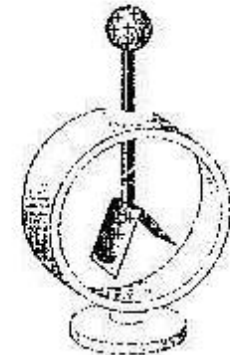
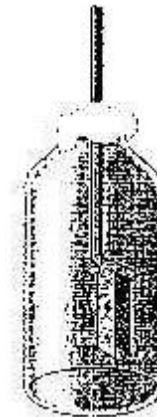
$\frac{Q}{2}$

$\frac{Q}{2}$

The distribution of charge on the surfaces of identical conductors. The charge divides evenly on the two spheres.

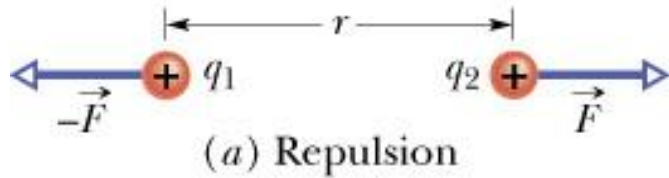


Charge tends to bunch up on the pointed regions of a conductor.



Electroscopes homemade and otherwise. No longer used in the laboratory, the electrostatic demonstrator is now primarily a teaching device. To make one, remove the thin aluminum foil from the wrapper on a stick of gum. Hang it on a thick wire, lower it into a bottle, and seal the bottle with wax or clay.

Coulomb's Law



Consider two charges q_1 and q_2 placed at a distance r . The two charges exert a force on each other that has the following characteristics:

1. The force acts along the line connecting the two charges.
2. The force is attractive for charges of opposite sign. The force is repulsive for charges of the same sign.
3. The magnitude of the force, known as Coulomb

force, is given by the equation $F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$.

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$

The constant ϵ_0 is known as the **permittivity constant** $\epsilon_0 = 8.85 \times 10^{-12} \text{ N} \cdot \text{m}^2 / \text{C}^2$.

The Coulomb force has the same form as Newton's gravitational force. The two differ in one aspect: The gravitational force is always attractive. Coulomb's force on the other hand can be either attractive or repulsive depending on the sign of the charges involved.

Units of Charge

The unit of charge in the SI unit system is the "Coulomb" (symbol C).

In principle we could use Coulomb's law for two equal charges q as follows:

Place the two charges at a distance $r = 1\text{m}$. $q = 1\text{ C}$ if $F = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\text{ N}$:

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \rightarrow F = \frac{1}{4 \times 3.14 \times 8.85 \times 10^{-12}} \frac{1^2}{1^2} = 8.99 \times 10^9\text{ N}$$

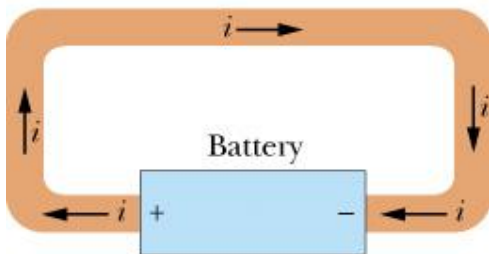
For practical reasons that have to do with the accuracy of the definition, the electric current is used instead. The electric current i in the circuit of the figure

is defined by the equation $i = \frac{dq}{dt}$, i.e., the amount of charge that flows

through any cross section of the wire per unit time. The unit of current in SI is the ampere (symbol A) and it can be defined very accurately.

If we solve the equation above for dq we get $dq = idt$.

Thus if a current $i = 1\text{A}$ flows through the circuit, a charge $q = 1\text{C}$ passes through any cross section of the wire in one second.



(b)

Problem.

A coulomb is a fairly large amount of charge. In fact, it is very difficult to assemble that amount of charge in an electrostatic arrangement without a breakdown or discharge.

- Calculate the number of electrons associated with a charge of -1C .
- Determine the mass of this number of electrons.

Solution.

$$\text{a) } n = \frac{-1\text{C}}{-1.6 \times 10^{-19} \text{ C/electron}} = 6.2 \times 10^{18} \text{ electron}$$

$$\text{b) } m = (1.67 \times 10^{-31} \text{ kg/electron}) \times (6.2 \times 10^{18} \text{ electron}) = 1 \times 10^{-12} \text{ kg}$$

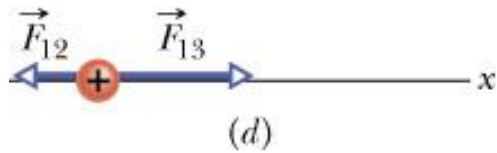
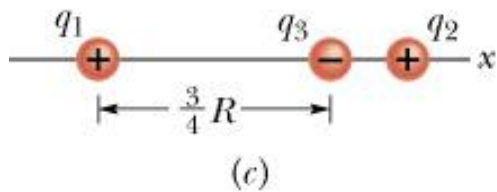
Problem.

The electric charge in one mole of protons is called **Faraday's constant**.

What is its numerical value?

Solution.

$$Q = N_A * e = (6.02 \times 10^{23}) \times (1.6 \times 10^{-19}\text{C}) = 9.632 \times 10^4\text{C}$$



Coulomb's Law and the Principle of Superposition

The net electric force exerted by a group of charges is equal to the vector sum of the contribution from each charge.

For example, the net force \vec{F}_1 exerted on q_1 by q_2 and q_3 is equal to $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$.

Here \vec{F}_{12} and \vec{F}_{13} are the forces exerted on q_1 by q_2 and q_3 , respectively.

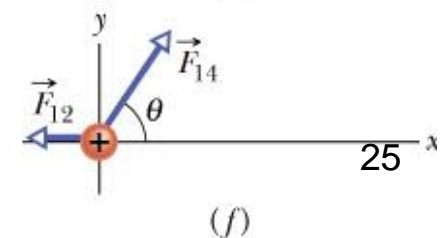
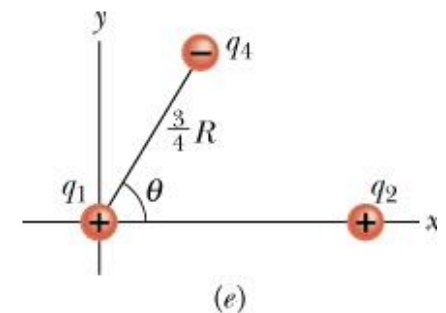
In general, the force exerted on q_1 by n charges is given by the equation

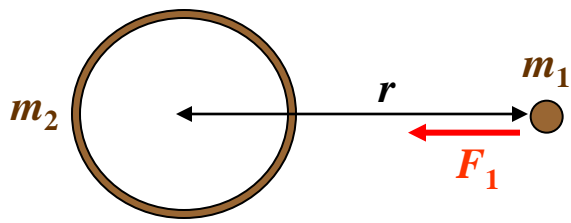
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots + \vec{F}_{1n} = \sum_{i=2}^n \vec{F}_{1i}$$

One must remember that \vec{F}_{12} , \vec{F}_{13} , ... are vectors and thus

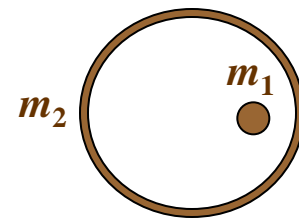
we must use vector addition. In the example of fig. *f* we have:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{14}$$

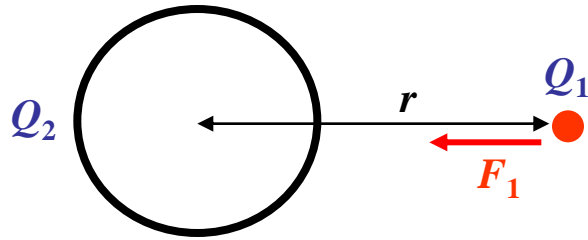




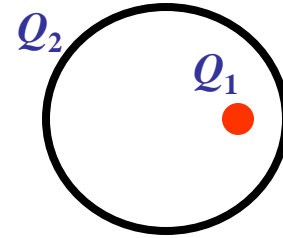
$$F_1 = G \frac{m_1 m_2}{r^2}$$



$$F = 0$$



$$F_1 = \frac{1}{4\pi\epsilon_0} \frac{|Q_1||Q_2|}{r^2}$$



$$F_1 = 0$$

The gravitational force that a uniform shell of mass m_2 exerts on a particle of mass m_1 that is outside the shell is given by the equation $F_1 = G \frac{m_1 m_2}{r^2}$.

It is as if the shell's mass m_2 were all concentrated at the shell center.

If m_1 is inside the shell, the net force exerted by m_2 is zero.

Because of the similarity between Newton's gravitational law and Coulomb's law, the same is true for the electric force exerted by a spherical shell of charge Q_2 on a point charge Q_1 . If Q_1 is outside the shell, then the force F_1 exerted by Q_2 is

$F_1 = \frac{1}{4\pi\epsilon_0} \frac{|Q_1||Q_2|}{r^2}$. If Q_1 is inside the shell, then the force $F_1 = 0$.

Problem. The electric force of attraction between an electron and a proton separated by a distance of 0.53×10^{-10} m is 8.2×10^{-8} N. What is the force if the separation is twice as large? Three times as large. Four times as large?

$$F = 1/(4\pi \epsilon_0) (q_1 \cdot q_2)/ r^2 = 8.2 \times 10^{-8} \text{ N}$$

$$F' = 1/(4\pi \epsilon_0) (q_1 \cdot q_2)/ (r')^2 = ?$$

$$\begin{aligned} r' = 2r: \quad F' &= 1/(4\pi \epsilon_0) (q_1 \cdot q_2)/ (r')^2 = 1/(4\pi \epsilon_0) (q_1 \cdot q_2)/ 4r^2 = \frac{1}{4} F \\ &= \\ &= \underline{2.05 \times 10^{-8} \text{ N}} \end{aligned}$$

$$\begin{aligned} r' = 3r: \quad F' &= 1/(4\pi \epsilon_0) (q_1 \cdot q_2)/ (r')^2 = 1/(4\pi \epsilon_0) (q_1 \cdot q_2)/ 9r^2 = \frac{1}{9} F \\ F &= \\ &= \underline{0.91 \times 10^{-8} \text{ N}} \end{aligned}$$

$$r' = 4r: \quad F' = \frac{1}{16} F = \underline{0.51 \times 10^{-8} \text{ N}}$$

Problem.

Suppose that two grains of dust of equal masses each have a single electron charge. What must be the masses of the grains if their gravitational attraction is to balance their electric repulsion?

$$F_{\text{grav}} = F_{\text{elec}} \quad ; \quad G m^2 / r^2 = 1 / (4\pi \epsilon_0) \quad e^2 / r^2$$

$$m = e \sqrt{[1 / (G 4\pi \epsilon_0)]} = (1.6 \times 10^{-19} \text{ C}) \sqrt{\frac{9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2}}$$
$$= (1.6 \times 10^{-19} \text{ C}) \times (1.16 \times 10^{10} \text{ kg/C}) = \underline{2 \times 10^{-9} \text{ kg}}$$

Problem.

Suppose that the two protons in the nucleus of a helium atom are at a distance of $2 \times 10^{-15} \text{ m}$ from each other. What is the magnitude of the electric force of repulsion that they exert on each other? What would be the acceleration of each if this were the only force acting on them? Treat the protons as point particles.

$$F = 1 / (4\pi \epsilon_0) \quad (q_1 \cdot q_2) / r^2 = (9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(1.6 \times 10^{-19} \text{ C}) \times (1.6 \times 10^{-19} \text{ C})}{(2 \times 10^{-15})^2 \text{ m}^2}$$
$$= 58 \text{ N}$$

$$a = F/m = 58 \text{ N} / 1.67 \times 10^{-27} \text{ kg} = \boxed{3.4 \times 10^{28} \text{ m/s}^2}$$

Problem. The electric charge flowing through an ordinary 115-Volt, 150 Watt light bulb is 1.3 C/s. How many electrons/second does this amount to?

1 electron has $1.6 \times 10^{-19} \text{ C}$. Therefore

$$1.3 \text{ C/s} = \left(\frac{1.3 \text{ C}}{1.6 \times 10^{-19} \text{ C/electron}} \right) / \text{s} =$$

$$= 8.125 \times 10^{18} \text{ electron/s}$$

Problem A small charge of $2 \times 10^{-6} \text{ C}$ is at the point $x=2 \text{ m}$, $y=0$ on the x axis. A second small charge of $-3 \times 10^{-6} \text{ C}$ is at the point $x=0$, $y=-3 \text{ m}$ on the y axis.
 1) What is the electric force that the first charge exerts on the second F_{12} ? 2) F_{21} ? Express the answers as vectors with x and y components.

$$\vec{F}_{12} = -\vec{F}_{21}$$

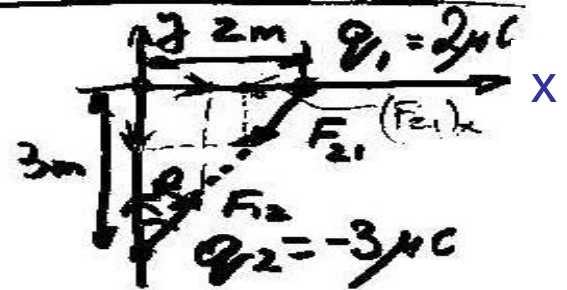
$$|F| = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = 9 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \times$$

$$\times \frac{(2 \times 10^{-6} \text{ C})(3 \times 10^{-6} \text{ C})}{(2 \text{ m})^2 + (3 \text{ m})^2} = 4.2 \times 10^{-3} \text{ N}$$

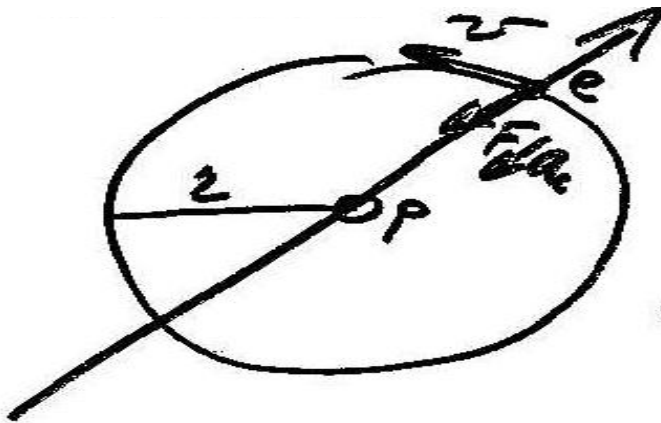
$$\tan \theta = \frac{2}{3}; \quad \theta = \arctan\left(\frac{2}{3}\right) = 33.7^\circ$$

$$(F_{12})_x = F \sin \theta = 2.3 \times 10^{-3} \text{ N}; \quad (F_{21})_x = -(F_{12})_x = -2.3 \times 10^{-3} \text{ N}$$

$$(F_{12})_y = F \cos \theta = 3.5 \times 10^{-3} \text{ N}; \quad (F_{21})_y = -(F_{12})_y = -3.5 \times 10^{-3} \text{ N}$$



Problem. Under the influence of the electric force of attraction, the electron in a hydrogen atom orbits around the proton on a circle of radius 0.53×10^{-10} m. What is the orbital speed? What is the orbital period?



Electric force

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 \cdot q_2}{r^2} =$$

$$= 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \frac{(1.6 \times 10^{-19})^2 \text{C}^2}{(0.53 \times 10^{-10})^2 \text{m}^2} =$$

$$= -8.2 \times 10^{-8} \text{ N}$$

Let "v" be the velocity of electron.
To remain in circular orbit, we need

$$m \overline{a_c} = \overline{F} ; \quad a_c = \frac{v^2}{r}$$

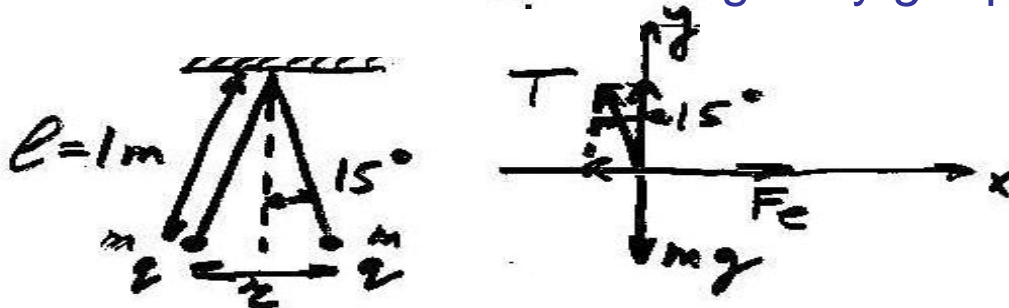
$$\Rightarrow -m \frac{v^2}{r} = -F ;$$

$$\Rightarrow v = \sqrt{\frac{F r}{m}} = \sqrt{\frac{(8.2 \times 10^{-8} \text{ N}) \times (0.53 \times 10^{-10} \text{ m})}{(9.1 \times 10^{-31} \text{ kg})}} =$$

$$= \boxed{2.2 \times 10^6 \text{ m/s}}$$

orbital period $T = \frac{2\pi r}{v} = \frac{2\pi (0.53 \times 10^{-10} \text{ m})}{2.2 \times 10^6 \text{ m/s}} = \boxed{1.5 \times 10^{-16} \text{ s}}$

Problem. Two Styrofoam balls each of mass 4 g, are hung from a common point in the ceiling by silk threads 1 m long. After being given identical charges, the balls repel each other and hang so that each thread makes an angle of 15° with the vertical. Find the charge given to each Styrofoam ball. The acceleration of gravity g equals 9.8 m/s^2 .



1) In a problem like this, carefully draw diagrams especially "free-body" diagrams as most important.

2) The electrical force of repulsion

$$F_E = \left(\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \right) \text{ N}, \text{ where } r = 2l \times \sin 15^\circ = 0.52 \text{ m}$$

3) Equilibrium condition $F_{net\ x} = 0$; $F_{net\ y} = 0$

$$x: F_E - T \sin 15^\circ = 0 \quad (i)$$

$$y: T \cos 15^\circ - mg = 0 \quad (ii) \quad \frac{(i)}{(ii)} \Rightarrow \frac{F_E}{mg} = \frac{T \sin 15^\circ}{T \cos 15^\circ} = \tan 15^\circ$$

$$\therefore F_E = mg \tan 15^\circ = (4 \times 10^{-3}) (9.8) \tan 15^\circ = 1.05 \times 10^{-2} \text{ N}$$

4) We already know that $F_E = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$, where $r = 0.52 \text{ m}$

$$\text{Solving for } q, q = \sqrt{F_E \cdot 4\pi\epsilon_0 \cdot r^2} = 5.5 \times 10^{-7} \text{ C}$$